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On the production of open strings from brane anti-brane annihilation

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Abstract: We investigate the leading contribution to open string production in the time dependent background of the Brane Anti-Brane. This is a 1-loop diagram and we use Boundary Conformal Field Theory (BCFT) techniques to study it. We show that the amplitude to a single open string naïvely diverges when one looks at it as an expansion in oscillator levels. Nevertheless, we show that once we sum over all oscillator levels we get a finite result. We also clarify where to perform the inverse Wick rotation in this kind of problems. This calculation could have important consequences for the theory of reheating in brane inflationary models.

Keywords: Tachyon Condensation, Cosmology of Theories beyond the SM.

Contents

1. Introduction and phenomenological motivation

Brane inflation is a powerful new paradigm in cosmology that might lead to interesting predictions [1 – 6]. If these predictions are observed experimentally they might open windows into string theory itself. We can put these predictions into two different types. First from the inflation potential, one can predict all the usual inflationary parameters measured in the cosmic microwave background. Althought interesting, it turns out that realistic models of brane inflation have enough freedom and parameters to accommodate (without much fine-tuning in some cases) various power spectrum that fits the data [7 – 14]. Of course a plethora of different models of inflation fits the data and so we must see these measurements not as an experimental test of string theory but rather as constraints as to where in the landscape our universe inflated.

Brane inflation also predicts how the universe should reheat. In the simplest models, inflation occurs when branes collide and annihilate. These features of brane inflation cannot be described by usual quantum field theory and we can therefore hope to have purely stringy phenomena coming out of it. The most famous prediction of this type is the production of cosmic strings [15 – 20] at the end of brane inflation. The production of cosmic strings can be shown to happen in $D\bar{D}$ annihilation from BCFT methods [21] or Boundary String Field Theory methods [22 – 24]. Their properties could shed interesting light into string theory [25, 26]. The phase transition at the end of brane inflation will not only produce

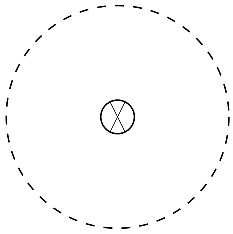

Figure 1: Disc diagram with insertion of a closed string vertex operator (this is of order g_s^0).

defects but also particles which lead to reheating. There has already been some study of reheating in the context of tachyon condensation for the brane anti-brane system [27 − 29]. And recently, a more thorough analysis of reheating in KKLMMT-type models [30] was performed in [31 − 33]. Additionally, a discussion of reheating in heterotic string theory models can be found in [34].

The starting point of these calculations is that D$\bar{\mathrm{D}}$ annihilates completely to non-relativistic massive closed strings [35, 36]. Then they subsequently decay to standard model radiation. Although this is certainly a legitimate starting point when there are no other branes around, it is not clear that it is correct when the brane annihilates with a stack of anti-branes. Indeed, one might expect that such a collision should excite the spectator branes as well as producing closed strings.

Motivated by these phenomenological implications we would like to calculate the rate of production of open strings on a spectator D-brane in the time-dependent background of a decaying brane anti-brane (D$\bar{\mathrm{D}}$) system. The result should be compared to the rate to massive closed strings already calculated by Liu, Lambert and Maldacena (LLM) [35]. In the LLM paper they found that the closed strings emitted are highly massive and localized to the plane of the D$\bar{\mathrm{D}}$ system. The amplitude therein goes like $\frac{1}{g_s} \times g_s = g_s^0$, the first factor from the disc, the second from the closed string vertex operator. The relevant calculation uses the boundary state calculations of [21, 37] with an interacting BCFT and the insertion of a single closed string vertex operator.

If another brane is added to the decaying system, the time-dependent background is expected to generate open string excitations. This would correspond to the first step of the reheating of the universe in D$\bar{\mathrm{D}}$ models of inflation, so it is important to estimate the rate and compare to the rate to closed strings.

The leading open string creating process is a one-loop effect. Therefore, in this paper, we will extend the work of LLM to 1-loop. To see that open string creation is a 1-loop effect, note that the DD$\bar{\mathrm{D}}$ system has states in the Chan-Paton matrix (see for example [38]):

$$\begin{pmatrix} \begin{pmatrix} A^{\bar{\mathrm{D}}} & \bar{T} \\ T & A^{\mathrm{D}} \end{pmatrix} & S^{\dagger} \\ S & A \end{pmatrix}$$

Note that at lowest level, S contains a W-type vector boson, and a tachyon, but this will be unimportant; we take S to indicate *any open string states* stretching between the decaying D$\bar{\mathrm{D}}$ system and the spectator brane. Similarly, A at lowest level would denote the gauge

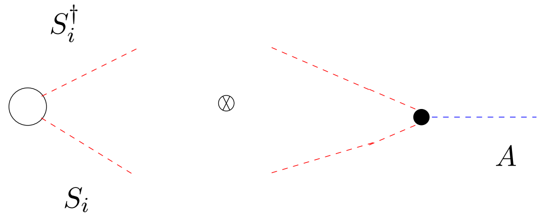

Figure 2: D$\bar{\text{D}}$ (large white circle) emitting the fields S and $S^\dagger$ that then annihilates on a spectator brane to produce a gauge field A.

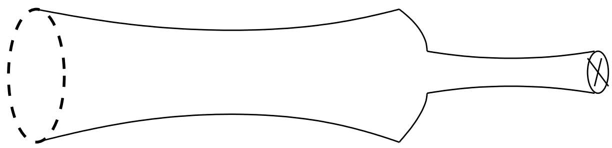

Figure 3: 1-point function on the cylinder where one end (dashed line) represents the D$\bar{\text{D}}$ time dependent source and the other end represents the spectator brane.

field on the remaining D-brane but here we take it to be *any open string states* living on the spectator brane. The states S cannot be in any final state since they have one end on the decaying system, so only A can be a final state. However, in order to produce A, S must be excited since there is no direct coupling between A and T which is the time-dependent source. Note that as for the decay to closed strings, the $A^{\mathrm{D},\bar{\mathrm{D}}}$ strings and their excitations are irrelevant and can be set to 0. The lowest order process to excite A then is:

As this is a 1 point function, on the spectator branes, the field A must be a gauge singlet. The corresponding string diagrams are disc amplitudes. When summed over all possible intermediate states, S_i, they lead to the annulus diagram with one boundary corresponding to the decaying D$\bar{\text{D}}$ system and the other boundary representing the spectator D-brane, with a vertex operator insertion corresponding to A. The string calculation is

$$\mathcal{A}(E) = \langle B| \frac{1}{L_0 + \tilde{L}_0} \mathcal{V} |N\rangle \ , \tag{1.1}$$

where $\langle B|$ is the boundary state of the brane anti-brane system, $\mathcal{V}$ represents the vertex operator of the state produced and $|N\rangle$ is the usual Neumann boundary state for the spectator brane.

This process then goes like $g_s^0 \times (g_s^{1/2}) = g_s^{1/2}$, where the first factor comes from the annulus, and the second from the open string vertex operator. That this is higher in string coupling naïvely implies that this contribution should be subleading compared to closed string production. On the other hand if one looks at pair production (which is now of order g_s), we are no longer limited to gauge singlet and we will therefore have an enhancement from the number of spectators branes. Indeed, the square of the amplitude for the 2-point function goes like g_s^2 but when we sum over all states we get equal contribution for all the states produced in the Chan-Panton matrix. For an SU(N) gauge theory we would get $N^2 - 1$ such states and therefore the controlling parameter is roughly $N^2 g_s^2$. It is then possible for sufficiently large N to imagine that the 2-point function would lead to a non negligible production of open strings.

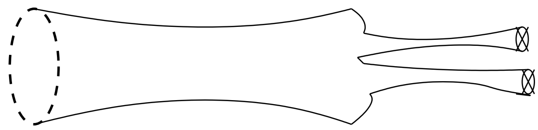

Figure 4: 2-point function on the cylinder where one end (dashed line) represents the D$\bar{\rm D}$ time dependent source and the other end represents the spectator brane.

In this paper we will concentrate on the one-point function as it is much simpler and exhibits the main properties and problems that one encounters also for the 2-point function. Nevertheless we will comment on the 2-point function later on as this is surely the most important diagram for open string production. As we will discuss later, there is a diagram at 1-loop that will contribute to closed string emission and it will also be enhanced by the same factor of N. Therefore, enhancement by factors of N is not necessarily an advantage of open strings production versus closed strings but it does show that g_s is not necessarily the controlling parameter.[1]

These are the motivations behind these 1-loop calculations but this paper will mostly address the computational techniques needed and the problems associated with them. Indeed, it appears that the 1-loop amplitude in the time dependent background of D$\bar{\rm D}$ is not well defined if one expands the amplitude in oscillator levels as we get terms that diverge at late time (as was noted in [39]). Historically these terms were thought to cause problems for the tree-level calculation [40] but [35] showed that an appropriate choice of gauge leads to a finite tree-level amplitude. In the rest of this paper we will show that these divergent terms do appear in the 1-loop amplitude and they cannot be gauged away. Nevertheless we argue that the amplitude is finite and that all the divergent terms come in such a way that they sum to a finite result. We argue this by truncating Sen's boundary state to an infinite subset of terms and doing so we get the following behavior for the 1-loop amplitude:

$$\mathcal{A}_{\rm 1pt}(E) \approx \frac{1}{\sinh \pi E} w(E)\,, \qquad \mathcal{A}_{\rm 2pts}(E) \approx \frac{1}{\sinh 2\pi E} w'(E)\,,$$

where $w(E)$ and $w\prime(E)$ are unknown polynomials. This form for the amplitude would imply that open string production is by itself non negligible but in general phase space consideration would still favor closed string emission over open string. We cannot exactly predict how much open strings versus closed strings will be produced without a specific knowledge of these polynomials but it appears that, due to phase space consideration (more phase space for closed strings), closed strings production will dominate in most cases.

In this paper we will first discuss more in depth how the problem with divergent terms arises, then we will look in detail at the calculation by first reviewing the BCFT techniques as well as the tree level calculation of [35]. We then look at a subset of terms contributing to the amplitude and show how these terms can be summed to something finite. Finally we will conjecture about the final form of the amplitude and discuss its implications for the theory of reheating.

[1]We thank Xingang Chen for pointing this to us.

2. Problematic divergent terms in Sen's boundary state

The goal of this paper is to explore the properties of the 1-loop amplitude in the time dependent background of $D\bar{D}$. Before going into details for any calculations, let us first describe the main problems that we will encounter.

First of all there is a technical difficulty as the vertex operators of physical open string states are naturally expressed in the Virasoro basis (α) as opposed to the brane anti-brane boundary state which is naturally expressed in terms of the SU(2) current algebra basis. This clash of basis leaves two main routes of attack to perform the calculation.

The first line of attack comes from the realization that the boundary states only excite open strings with integer values of momentum. This allows us to write the vertex operator either as a product of SU(2) current operators or as a product of fermions. This way the full amplitude can be evaluated for specific values of momenta and then we can sum over all momenta. This calculation turns out to be well defined but technically intractable as the terms in the sum get quickly out of control for large momentum. We therefore merely present the basic idea in the appendix.

The second line of attack is to expand the boundary state in the α basis and calculate the amplitude as a series in power of oscillator levels

$$\begin{aligned}|B\rangle &= B_{0,0}\,|0\rangle + B_{1,1}\alpha^0_{-1}\tilde{\alpha}^0_{-1}\,|0\rangle + \cdots\,,\\ \mathcal{A} &= \mathcal{A}_{0,0} + \mathcal{A}_{1,1} + \cdots\,.\end{aligned}$$

This method is instructive as it shows the subtleties of the Wick rotation in such a system. We show that this series appears to be badly behaved as individual terms at level 2 and beyond seems to diverge at late time. As we will see, choosing a special gauge is not going to solve the problem for the 1-loop calculation as it does for the tree level. On the other hand, by using a subset of terms in the boundary state expansion to all levels (namely all terms of the form $\alpha^0_{-n}\tilde{\alpha}^0_{-n}$) we can show that part of these divergent pieces lead to a finite contribution once you sum over *all momenta and all oscillator levels* before Wick rotating. This essentially extends the original work of Sen and it is the main result of this paper. Although interesting, this does not tell us very much about the amplitude as we have only chosen a subset of all the strings running in the loop. Still, it does give tantalizing hints that the amplitude should be finite.

3. Rolling tachyon boundary conformal field theory

The time-dependent BCFT representing a homogeneously evolving tachyon on a D-brane in bosonic string theory was introduced by Sen [21]. To be a solution to classical open string theory, the insertion on the worldsheet boundary must be marginal, and independent of oscillators to correspond to the tachyon. One possible deformation (considered by Sen) is therefore $\lambda \cosh X^0/\sqrt{\alpha'}$ (in the remainder of this paper, we shall work in units where $\alpha' = 1$)

$$S = -\frac{1}{2\pi}\int d^2z\partial_z X^0\partial_{\bar{z}}X^0 + \lambda\int d\tau\cosh X^0\,.$$

By Wick rotating $X^0 \to iX^0$, one gets the celebrated Sine-Gordon action for which the boundary state is well known [41]. It turns out to be a simple SU(2) rotation of the Neumann boundary state

$$
\begin{aligned}
|B\rangle &= e^{i\theta^a J^a}\,|N\rangle \\
|B\rangle &= \sum_j \sum_{m=-j}^{j} D^j_{m,-m}(R)\,|j,m,m\rangle\rangle\,.
\end{aligned}
$$

where R is a SU(2) rotation matrix and J^a are the SU(2) current operator (see appendix).

This tachyon profile is often called the full S-brane and it represents finely tuned incoming radiation creating the D$\bar{\mathrm{D}}$ system, followed by its decay back into radiation. Of course the decay is the only physically relevant part. The tachyon profile $\tilde{\lambda}e^{X^0}$ ($\frac{1}{2}$ S-brane) captures only the decay part and is therefore what we will use for the rest of this paper.

The boundary state for the $\frac{1}{2}$ S-brane is a simple generalization of the full S-brane case where the rotation matrix is now an element of SL$(2,\mathcal{C})$. The explicit form is (with $\tilde{\lambda} = 2\pi\lambda$) [42]:

$$
|B\rangle \equiv \sum_{j\in\frac{1}{2}\mathbb{Z}^+} \sum_{m\geq 0}^{j} \binom{j+m}{2m} (-\tilde{\lambda})^{2m}\,|j,m,m\rangle\rangle\ . \tag{3.1}
$$

Sen calculated those parts of the corresponding boundary state which are oscillator-free, and proportional to $\alpha^0_{-1}\tilde{\alpha}^0_{-1}$, in the euclidean theory,

$$
|B\rangle = \int dE\,\{f_{\mathrm{Eu}}(E) + g_{\mathrm{Eu}}(E)\alpha^0_{-1}\tilde{\alpha}^0_{-1} + \ldots\}\,|E\rangle\ . \tag{3.2}
$$

In the Wick rotated theory, f_{Eu} and g_{Eu} are calculated either in a perturbative expansion of the boundary interaction or by using the SU(2) structure of the system,

$$
\begin{aligned}
f_{\mathrm{Eu}}(E) &= \langle E|B\rangle_{\mathrm{Eu}} = \sum_{n=0}^{\infty}(-\tilde{\lambda})^n 2\pi\delta(n-E)\,, \\
g_{\mathrm{Eu}}(E) &= \langle E|\,\alpha^0_1\tilde{\alpha}^0_1\,|B\rangle_{\mathrm{Eu}} = \sum_{n=1}^{\infty}(-\tilde{\lambda})^n 2\pi\delta(n-E) = f_{\mathrm{Eu}}(E) - 2\pi\delta(E)\,.
\end{aligned}
$$

An inverse Wick rotation of these quantities would be somewhat ill defined. Sen argues that it is sensible only to inverse Wick rotate the appropriate, physically meaningful quantities: $f_{\mathrm{Eu}}(\tau)$ and $g_{\mathrm{Eu}}(\tau)$, the Fourier transform of f and g, which have meaning as components of the stress-energy tensor of the decaying brane system [21, 42]. Consequently,

$$
\begin{aligned}
f_{\mathrm{Eu}}(\tau) = \int \frac{dE}{2\pi} e^{i\tau E} f_{\mathrm{Eu}}(E) &= \frac{1}{1+\tilde{\lambda}e^{i\tau}} \\
\to f(t) &= \frac{1}{1+\tilde{\lambda}e^{t}}\,.
\end{aligned}
$$

Similarly,

$$
\to g(t) = 2\pi\left[\frac{1}{1+\tilde{\lambda}e^{t}} - 1\right],
$$

where we have done an inverse Wick rotation ($\tau = -it$). This procedure defines well the stress-energy tensor and the boundary state, because f and g have well-defined Fourier transforms,

$$\begin{aligned} f(E) &= \frac{i\pi\tilde{\lambda}^{-iE}}{\sinh(\pi E)}\,, \\ g(E) &= f(E) - 2\pi\delta(E)\,. \end{aligned}$$

However, when considering other states that should appear in (3.2), this is no longer true; for instance, [40, 43] calculate the coefficients of $\alpha^0_{-2}\tilde{\alpha}^0_{-2}$ and $(\alpha^0_{-1})^2\tilde{\alpha}^0_{-2}$ in $|B\rangle$ to contain terms like

$$\sim \int dt\; e^{t+iEt}\,. \tag{3.3}$$

In [40], this was interpreted as the coupling of the decaying brane system to massive closed strings diverging at late times. However, [35] calculate the rate of generation of closed strings from a decaying brane, and showed that the rate to an individual closed string does not diverge.

The leading diagram that contributes to the production of closed strings is the disk diagram (see figure 1). The corresponding calculation in the euclidean theory is

$$\mathcal{A}_{\text{disk}}(E) = \langle 0|\,\mathcal{V}_c\,|B\rangle\ . \tag{3.4}$$

It was argued in [44, 45], that one can choose a gauge for the *physical* states such that there are no time-like oscillators in the vertex operator besides the momentum part,[2]

$$\mathcal{V}_c = e^{iEX^0}\mathcal{V}_{\text{sp}}\,. \tag{3.5}$$

The spatial part of the amplitude then turns out to be simply a phase and the timelike part simply picks up the part of the boundary that contains no oscillators. Hence we find that the amplitude is

$$\begin{aligned} \mathcal{A}_{\text{disk}}(E) &= \langle E|B\rangle\ , \\ &= 2\pi\sum_n(-\tilde{\lambda})^n\delta(E-n) = f_{\text{Euc}}(E)\,. \end{aligned} \tag{3.6}$$

This is the amplitude (up to a phase) in the euclidean theory. By the same procedure as before we get in the lorenztian theory:

$$\begin{aligned} \mathcal{A}_{\text{disk}}(E) &= \int dt\frac{e^{iEt}}{1+\tilde{\lambda}e^t} \\ &= \pi i\frac{e^{-iE\log\tilde{\lambda}}}{\sinh\pi E}\,. \end{aligned} \tag{3.7}$$

[2]Note that this gauge choice only applies to physical closed and open string states and it does not affect the boundary states, we are grateful to Nicolas Jones for clarifying this point.

From this, we see that these divergent pieces do not show up in the tree-level calculation as one can choose a gauge for the emitted vertex where there are no oscillators in the timelike direction. By doing so, the only part of the boundary state contributing to the amplitude is the zero mode part $f(E)$. Loosely speaking, one could say that these divergent terms can be 'gauged away' in the tree level calculation. This also means that $f(E)$ is sufficient to completely specify every physical amplitude of closed strings at tree level.

At one-loop one expects a different outcome. Indeed, the full boundary state is needed as it is to be projected onto a Neumann boundary state containing all levels of oscillators. Choosing a special gauge for the emitted vertex will not prevent these terms from appearing when we trace over the whole open string spectrum. Nevertheless, it would be extremely surprising if the amplitude of the brane anti-brane system to decay to an *individual* open string was divergent.

Another important issue that we encounter while going to a 1-loop analysis of tachyon condensation is that it is ambiguous where we should perform the inverse Wick rotation to go back to a lorentzian signature. As was pointed out recently [46, 42], since terms like (3.3) are ill-defined, there must be some *physical prescription to define them*. After we inverse Wick rotate to the lorentzian frame, this could be given by the condition of conformal invariance of the boundary state:

$$(L_n - \tilde{L}_{-n})\,|B\rangle = 0\,, \quad \forall n \in \mathbb{Z}\,. \tag{3.8}$$

Imposing this condition gives a way to define (3.3), via

$$\begin{aligned} \langle E|\,(\alpha_1^0)^2\tilde{\alpha}_2^0\,|B\rangle \;&=\; \frac{1}{E}\,\langle E|\,L_1\alpha_1^0\tilde{\alpha}_2^0\,|B\rangle = \ldots \\ \Rightarrow \left(1-\frac{1}{E\cdot E}\right)\langle E|\,(\alpha_1^0)^2\tilde{\alpha}_2^0\,|B\rangle \;&=\; 0\,, \end{aligned}$$

giving for $E \cdot E \neq 0, 1$, $\langle E|\,(\alpha_1^0)^2\tilde{\alpha}_2^0\,|B\rangle = 0$. For the *timelike system* we are considering, $E \cdot E = -E^2 \leq 0$, so as long as $E \neq 0$, (3.3) can be thus defined to be 0. As noted in [46, 42], this example reflects the fact that the $c = 1$ timelike CFT has no null vectors (the $c = 1$ Kac-Moody determinant is nowhere vanishing for $h < 0$). In this case it is therefore sufficient to consider the boundary state of Ishibashi states built over highest weight states of energy E, which captures all the necessary information of the timelike system:

$$|B\rangle = \int_0^\infty dE\; f(E)\,|E\rangle\!\rangle\;, \qquad |E\rangle\!\rangle = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}^0\tilde{\alpha}_{-n}^0\right]|E\rangle\;. \tag{3.9}$$

That is, the oscillator expansion of the boundary state is just that of a free timelike boundary state with Neumann boundary conditions. This reproduces f and higher diagonal states correctly (only for $E \neq 0$; terms at $E = 0$ are important in, for instance the construction of the stress-energy tensor of the system [21, 47], and are omitted from $|B\rangle$ here), and sets the coefficients of all off-diagonal states such as (3.3) to zero.

This approach is correct for tree-level calculations and it is perhaps the clearest way of understanding why these divergent terms do not appear in the calculation of [35]. On the other hand the boundary state (3.9) *should not* be used for 1-loop calculation.

Sen's prescription is to inverse Wick rotate only the physically meaningful quantities. The boundary state is physically meaningful because when projected on the closed strings vacuum it measures the 1-point tree level amplitude of closed strings from the decaying system. However, this is a trick and we need to remember that the inverse Wick rotated boundary state is only physically meaningful once it is projected on the vacuum. For the 1-loop amplitude, we do not project the boundary state on the vacuum and we should therefore not use the lorenztian boundary state (and its truncated but finite version (3.9)). Indeed, the correct calculation is to *stay until the very end in the euclidean frame* and then inverse Wick rotate the final result. Doing so, we will modify the sum leading to $f(x_0)$ and $g(x_0)$ and we will get *different* time dependence than what one gets at tree level.

This point is very important so let us elaborate more on the issue. It is quite clear that Sen's boundary state includes more than the simple finite diagonal terms [48]

$$|B\rangle = e^{\sum_{n>0}\frac{1}{n}\alpha^0_{-n}\alpha^0_{-n}} f(X^0)\,|0\rangle + \left|\tilde{B}\right\rangle .$$

Now this boundary state is physically meaningful as it measures the 1-point function on the disc so we can Wick rotate it. As we do so we find ill-defined terms (3.3). One can see that these terms are not a problems in two different ways. First one can see that in the lorentzian theory, we can define these ill-defined terms to be 0 by using conformal invariance. Secondly, we can simply choose a gauge for the vertex operators such that there are no oscillators in the time-like direction. It is an interesting question as to whether the two methods are related. Both ways fail when we try to calculate 1-loop amplitudes. Indeed choosing a special gauge (although probably useful for computational purposes) will not help as we project on a Neumann boundary state which will pick up these terms anyway. Furthermore, we cannot use the lorenztian boundary state as we need to do both the integral over moduli space 's' and trace over all the closed strings exchanged between the branes. Each step only converges if we do an analytic continuation to euclidean space forcing us to use the extra pieces of Sen's boundary state. At this point, we have a legitimate question to answer: is the 1-loop amplitude finite?

As we will argue in the following section, *we need to use the euclidean boundary state and inverse Wick rotate the finite result*, yet the answer is finite as the integral over the moduli 's' together with summing over all levels conspire to sum all these divergent terms to something finite.

4. Expanding the 1-loop amplitude in oscillator levels

Now, as explained before, the ultimate goal is to evaluate the 1-loop amplitude in superstring theory to any open string states. This is very hard and so we will first simplify to the simplest possible calculation. We will work in the bosonic theory and calculate the amplitude only to the open string tachyon. Even then, we will further simplify by only considering part of the boundary state $|B\rangle$. We will first disregard the spatial part of the boundary state (and disregard the tachyon coming from it which is an artifact of bosonic string theory) as well as expand the boundary state in oscillator levels. Doing so, our

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answer is not trustable to be a good approximation but the goal of this paper is more conceptual then quantitative. We hope to convince the reader that the dangerous terms coming from the time-like part of the boundary state do not lead to infinities.

We will calculate the amplitude (1.1) in the euclidean theory for the simplest state where it becomes (we calculate the complex conjugate for computational convenience):

$$\mathcal{A}_{\rm Eu}(E) = \int dx_0 \, \langle N| \, e^{-iEX^0} \frac{1}{L_0 + \tilde{L}_0} \, |B\rangle \ ,$$
$$\mathcal{A}_{\rm Eu}(E) = \int ds \int dx_0 \, \langle N| \, e^{-iEX^0} q^{L_0} \, |B\rangle \ ,$$

where $q = e^{-2s}$ and we have used the conformal condition on $|B\rangle$,

$$(L_n - \tilde{L}_{-n}) \, |B\rangle = 0 \, , \qquad \forall n \in \mathbb{Z} \, . \tag{4.1}$$

Note that no analytic continuation is needed in the last step as $E \cdot E = E^2 > 0$ and the integral over 's' converges. Now Sen's prescription tells us that we should inverse Wick rotate back only the fourier transform of $\mathcal{A}_{\rm Eu}(E)$

$$\mathcal{A}_{\rm Eu}(\tau) = \int \frac{dE}{2\pi} \int dx_0 \int ds e^{iE\tau} \, \langle N| \, e^{-iEX^0} q^{L_0} \, |B\rangle \ . \tag{4.2}$$

The final answer in the lorenztian frame is obtained after we inverse Wick rotate $\tau = -it$ and inverse Fourier transform

$$\mathcal{A}_{\rm Lor}(E) = \int dt e^{iEt} \mathcal{A}_{\rm Lor}(t) \, . \tag{4.3}$$

Note that the sign of the exponential is the same in both cases since we have done a Wick rotation. As a first exercise, let us evaluate explicitly the one-loop amplitude as an expansion in terms of oscillator levels

$$\left\langle :e^{-iEX^0}: \right\rangle = \int dx^0 \int ds \, \langle N| \, :e^{-iEX^0}: q^{p^2/4} Q^{L_0^{\rm osc}} \, |B\rangle \ .$$

Here, we have arbitrarily made a distinction between oscillators and momentum ($Q = q = e^{-2s}$). The goal will be to evaluate this amplitude as a series in Q. There is no good reason to expect Q to be a good expansion parameter, it is just that the calculation is tractable this way.

The boundary state for the rolling tachyon was worked out up to level (3,3) in [43] (and up to level (2,2) in [40])

$$|B\rangle = B^{(0;0} \, |0\rangle + B^{(1;1)} \alpha^0_{-1} \tilde{\alpha}^0_{-1} \, |0\rangle + \frac{1}{\sqrt{2}} B^{(1,1;2)} (\alpha^0_{-1})^2 \tilde{\alpha}^0_{-2} \, |0\rangle + \cdots \, .$$

We reproduce here table 4 of [43] where $\tilde{g} = -\tilde{\lambda} e^{ix_0}$ and $f = \sum_{n=0}^{\infty} \tilde{g}^n$.

The first thing to note in this table is that every possible left/right symmetric state (equal level on both side) contributes in the boundary state contrary to the usual Neumann boundary state where only left/right identical states contribute. The function f is common

$(\sigma;\tilde{\sigma})$	$B^{(\sigma;\tilde{\sigma})}$
$(0;0)$	f
$(1;1)$	$f-2$
$(2;2)$	$f-2+\tilde{g}$
$(1^2;1^2)$	$f+\tilde{g}$
$(1^2;2)$	$-\sqrt{2}\tilde{g}$
$(3;3)$	$f-2+\frac{4}{3}\tilde{g}-\frac{2}{3}\tilde{g}$
$(2,1;2,1)$	$f+\tilde{g}-2\tilde{g}^2$
$(1^3;1^3)$	$f-2+\frac{2}{3}\tilde{g}-\frac{4}{3}\tilde{g}^2$
$(2,1;1^3)$	$\sqrt{\frac{2}{3}}[\tilde{g}+2\tilde{g}^2]$
$(2,1;3)$	$\frac{2}{\sqrt{3}}[-\tilde{g}+\tilde{g}^2]$
$(3;1^3)$	$\frac{2\sqrt{2}}{3}[-\tilde{g}-\tilde{g}^2]$

Table 1: Boundary state coefficients up to level (3,3).

to most terms and leads to a finite contribution to the amplitude after we perform the sum in the euclidean theory. On the other hand, terms like $\tilde{g}$ will exponentially diverge at late time after Wick rotation. As this can hardly be made sense of, it must be they that somehow all conspire to sum to something finite.

We will get hints that this will happen by using a closed formula found in [43] for all terms of the form $\alpha^0_{-N}\tilde{\alpha}^0_{-N}$

$$B^{(N;N)} = f - \frac{2}{N}\sum_{n=0}^{N-1}(N-n)\tilde{g}^n\,. \tag{4.4}$$

Before tackling this infinite subset, let us see in more details how the amplitude behaves for the zero mode part where $|B\rangle = f\,|0\rangle$. We can get the zero mode part of the amplitude very simply:

$$\mathcal{A}_{0,0}(E) = \int dx_0 \int ds \sum_{n=1}^{\infty}(-\tilde{\lambda})^n e^{ix_0(n-E)} q^{n^2/4}\,.$$

In the last expression we got rid of the unphysical divergent $\langle 0|\, q^{L_0}\,|0\rangle$. We then fourier transform as explained before and integrate over x_0 and E

$$\begin{aligned}
\mathcal{A}_{0,0}(\tau) &= \int ds \sum_{n=1}^{\infty}(-\tilde{\lambda})^n e^{in\tau} q^{n^2/4}\,,\\
\mathcal{A}_{0,0}(\tau) &= 2\sum_{n=1}^{\infty}\frac{(-\tilde{\lambda}e^{i\tau})^n}{n^2}\,,\\
&= 2\mathrm{Li}_2(-\tilde{\lambda}e^{i\tau})\,,
\end{aligned}$$

where $\mathrm{Li}_2(z)$ is the polylogarithm function. We can now Wick rotate $\tau = -it$ and fourier transform back:

$$\mathcal{A}_{0,0}(E) = 2\int dt e^{iEt}\mathrm{Li}_2(-\tilde{\lambda}e^{t})\,.$$

The following equation relating the polylogarithm to the Fermi-Dirac distribution is very useful:

$$\mathrm{Li}_{(1+s)}(-e^{\mu}) = \frac{-1}{\Gamma(s+1)} \int_0^{\infty} \frac{k^s dk}{e^{k-\mu}+1} \,. \tag{4.5}$$

Using this, we can write the amplitude in a way very similar to [35], and the integral over time leads to a very similar result

$$\begin{aligned} \mathcal{A}_{0,0}(E) &= -2 \int_0^{\infty} dk \int dt \frac{k e^{iEt}}{\frac{1}{\tilde{\lambda}} e^k e^{-t} + 1} \,, \\ \mathcal{A}_{0,0}(E) &= -2 \int_0^{\infty} dk k e^{iEk} e^{iE\log(\tilde{\lambda})} \frac{\pi i}{\sinh(\pi E)} \,, \\ &= -e^{iE\log(\tilde{\lambda})} \frac{2\pi i}{\sinh(\pi E) E^2} \,. \end{aligned} \tag{4.6}$$

At this stage we have what we expected, this is the same as the tree level modified by a factor of $1/E^2$ coming from the closed string propagator. One can then generalize to higher levels and we will do so in more details in the following section.

Two complications arise when one look at this expansion to higher levels. The first is that one finds generically some hypergeometric functions that are, in general, difficult to integrate over time. This is only a technical problem as the integrand has no divergences other than poles.

The second complication is more important and the main source of worry coming from this amplitude. It is the fact that at level 2 or higher we start getting divergent terms that cannot be integrated of the form (3.3). We will now show how these terms can be summed into something finite.

5. Partial summation

At this point, our approximation is clearly unsatisfying. Here we will upgrade the approximation by doing a partial summation over all levels. We will use the fact that we know the boundary state coefficient of the form (4.4) to all levels to compute the amplitude for all this terms. So we take the boundary state to be of the form:

$$\begin{aligned} |B\rangle &= \left(B^{(0;0)} + \textstyle\sum_{N=1}^{\infty} \frac{1}{\sqrt{N}} B^{(N,N)} \alpha^0_{-N} \tilde{\alpha}^0_{-N} \right) |0\rangle \,, \\ B^{(N;N)} &= \sum_{n=0}^{\infty} \tilde{g}^n - \frac{2}{N} \sum_{n=0}^{N-1} (N-n) \tilde{g}^n \,. \end{aligned} \tag{5.1}$$

To evaluate this we will need to expand the vertex operator and the boundary state in the α^0 basis

$$e^{-iEX^0} = e^{-iE(x_0 + i\sqrt{2}\sum_{n>0} \frac{1}{n}(\alpha^0_n + \tilde{\alpha}^0_n))} = e^{-iEx_0} \left(1 + 2E^2 \sum_{n>0}^{\infty} \frac{1}{n^2} \alpha^0_n \tilde{\alpha}^0_n + \dots \right), \tag{5.2}$$

$$\langle N| = e^{-\sum_{n>0} \frac{1}{n} \alpha^0_n \tilde{\alpha}^0_n} = 1 - \sum_{n>0}^{\infty} \frac{1}{n} \alpha^0_n \tilde{\alpha}^0_n + \dots \,, \tag{5.3}$$

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where we have used the Neumann boundary condition ($(\alpha_n + \tilde{\alpha}_{-n})\,|N\rangle\!\rangle = 0$) for the vertex operator and we have also use the fact that $\eta_0^0 = +1$ in the euclidean theory for the Neumann boundary state. Commuting all the α^0 we get

$$\langle e^{-iEX_0}\rangle = e^{-iEx_0}\Bigg(\sum_{n=0}^{\infty}\tilde{g}^n q^{n^2/4} + \sum_{N=1}^{\infty}\sqrt{N}\left(\frac{2E^2}{N}-1\right)q^N \times \\ \times\left[\sum_{n=0}^{\infty}\tilde{g}^n q^{n^2/4} - \frac{2}{N}\sum_{n=0}^{N-1}(N-n)\tilde{g}^n q^{n^2/4}\right]\Bigg).$$

Then we take the fourier transform and intregate over x_0, E and s (in this order)

$$\mathcal{A}(\tau) = \mathcal{A}_{0,0}(\tau) + \sum_{N=1}^{\infty}\frac{1}{\sqrt{N}}\left[\frac{1}{2}\sum_{n=0}^{\infty}\frac{2n^2-N}{n^2+4N}z^n - \frac{1}{N}\sum_{n=0}^{N-1}(N-n)\frac{2n^2-N}{n^2+4N}z^n\right], \tag{5.4}$$

where $z = -\tilde{\lambda}e^{i\tau}$. This includes $\mathcal{A}_{0,0}$ calculated previously. The two sums over 'n' can be done but the answer is quite messy as one gets many different terms involving hypergeometric function of the type ${}_2F_1(1, -2i\sqrt{N}, 1-2i\sqrt{N}, z)$. The $\sqrt{N}$ makes the sum over N intractable analytically. These terms are not really worrisome but difficult to analyze to the end. As we are already considering only a subset of terms contributing to the amplitude, there is no reasons to even bother with them and so we will just drop them. Keeping only the part of the sum that can be done analytically:

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{2n^2-N}{n^2+4N}z^n &= \frac{2}{1-z}+\cdots, \\ \sum_{n=0}^{N-1}(N-n)\frac{2n^2-N}{n^2+4N}z^n &= \frac{-2z(1-z^N)}{(1-z)^2}+\frac{2N}{1-z}+\cdots.\end{aligned} \tag{5.5}$$

And, in this approximation the amplitude is

$$\mathcal{A}(\tau) \approx \mathcal{A}_{0,0}(\tau) + 4\sum_{N=1}^{\infty}\frac{1}{\sqrt{N}}\left[\frac{-1}{1+\tilde{\lambda}e^{i\tau}} + \frac{2}{N}\left(\frac{\tilde{\lambda}e^{i\tau}}{(1+\tilde{\lambda}e^{i\tau})^2} + \frac{(-\tilde{\lambda}e^{i\tau})^{N+1}}{(1-\tilde{\lambda}e^{i\tau})^2}\right)\right]. \tag{5.6}$$

The first term is divergent but this is an artifice of our approximation (5.5) as we expect that there should be other N dependent terms and the sum over N would then give a number. For the moment we simply regulate this sum using the Riemann zeta regulation. The second term in the sum is the source of all the worries mentioned at the beginning of this paper. For level 1, it is finite but for higher levels each term individually would diverge after inverse Wick rotation. But here comes the important point: the sum over oscillator levels is convergent before inverse Wick rotation, yielding a finite result for the amplitude

$$\mathcal{A}_{\text{partial}}(t) \approx Li_2(-\tilde{\lambda}e^t) + 4\Bigg[-\xi\left(\frac{1}{2}\right)\frac{1}{1+\tilde{\lambda}e^t} - \\ - 2\xi\left(\frac{3}{2}\right)\frac{-\tilde{\lambda}e^t}{(1+\tilde{\lambda}e^t)^2} + 2\frac{\tilde{\lambda}e^t}{(1+\tilde{\lambda}e^t)^2}Li_{3/2}(-\tilde{\lambda}e^t)\Bigg]. \tag{5.7}$$

Now we need to take the inverse Fourier transform. Doing so we see that each integral is convergent as we have more power of e^t in the denominator than in the numerator. Each integral gives a factor of $1/\sinh\pi E$. The last integral is a little more involved and we need to do a numerical integration for the dummy variable. It is simple to show(for large E) that one gets the following form for the amplitude:

$$\mathcal{A}_{\text{partial}}(E) \approx \frac{\pi i e^{-iE\ln\tilde{\lambda}}}{\sinh\pi E} w(E)\,, \tag{5.8}$$

where $w(E)$ is a polynomial. The answer is independent of λ up to an irrelevant phase as it must be for the $\frac{1}{2}$ S-brane. Now clearly, we cannot take $\mathcal{A}_{\text{partial}}$ too seriously as we calculated it by truncating the boundary state. By doing so, we have broken the SU(2) symmetry and we probably lost many other important contributions to the amplitude. Nevertheless, we see here that this part of the amplitude is finite after *we sum over both momentum and oscillator levels* generalizing the methods and results of Sen to 1-loop. A similar conclusion was obtained previously by Okuyama [49] for some specific vacuum amplitude using different techniques.[3]

This conclusion is expected to be true from T-duality since there should be no difference in our treatment of winding number and oscillator levels. This gives good hints that the worrisome terms in Sen's boundary state do not lead to inconsistencies. Unfortunately, without a complete expansion of the boundary state into the 'α' basis, it is impossible to push this calculation further. In the appendix we explain how one can use the SU(2) symmetry to perform the calculation.

6. Conjecture on the 2-point 1-loop amplitude

Finally, let us mention that it is quite unsatisfying that we have still such a limited knowledge of what D-branes decays into. It is important to realize that the common statement that D-branes decay completely into closed strings radiation is true only if there are no other D-branes around (and small g_s). The first correction to this behavior comes from the 1-loop open string production that we have investigated in this paper. There will be further corrections of order g_s (see figure 5).

Of all these additional diagrams, the two-point function of open strings looks the most interesting. Pair production of open strings might be energetically favorable over coherent production in certain cases. Here let us analyze the basic property of this process. Firstly, pair production in a time dependent background is different then the coherent production of the 1-point 1-loop amplitude and the 1-point tree level calculation that we have discussed in this paper. Indeed, for the latter two processes the out state is a coherent state and one can calculate the total number of particles produced as well as the total energy emitted to

[3]We are grateful to Kazumi Okuyama for bringing this to our attention.

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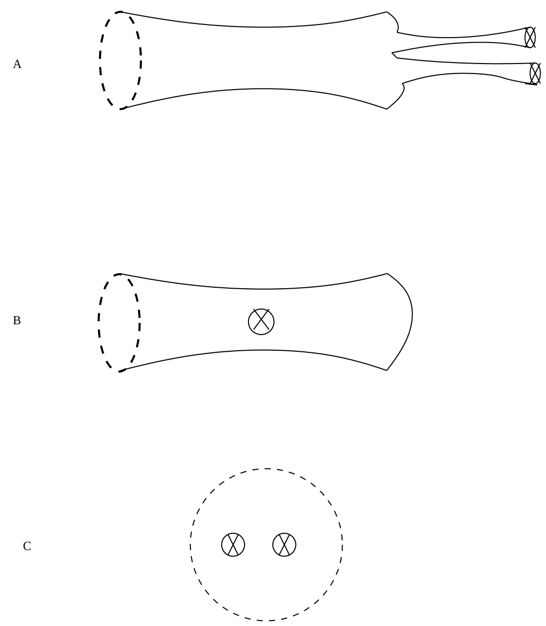

Figure 5: Next to next leading order correction to brane decays.

be [35]:

$$\frac{\bar{N}}{V_p} = \sum_s \frac{1}{2E_s}|\mathcal{A}_{\rm s}|^2\,, \tag{6.1}$$

$$\frac{\bar{E}}{V_p} = \sum_s \frac{1}{2}|\mathcal{A}_{\rm s}|^2\,. \tag{6.2}$$

On the other hand pair production lead to a *squeezed state* . Pair production of particles in a time dependent background has been studied extensively in the literature (see [50] for a review on the subject). The out state is:

$$|0\rangle^* = \prod_{\mathbf{k}} \left[\mathcal{N}_{\mathbf{k}} \exp\left(-\frac{\alpha_{\mathbf{k}}}{2} a^\dagger_{\mathbf{k}} a^\dagger_{-\mathbf{k}}\right)\right]|0\rangle\ , \qquad |\mathcal{N}_{\mathbf{k}}|^2 = |1-|\alpha_{\mathbf{k}}|^2|\,, \tag{6.3}$$

where $a^\dagger_{\mathbf{k}}$ is the spacetime creation operator for a bosonic state of momentum $\mathbf{k}$. The two-point amplitude for bosonic particles (out states) which couple to the time-dependent background is

$$\mathcal{A}_{\mathbf{k}} \equiv \langle\phi(\mathbf{k})\phi(-\mathbf{k})\rangle^* = \langle\mathbf{k},-\mathbf{k}|0\rangle^* = \frac{\langle 0|\sqrt{2E_{\mathbf{k}}}a_{\mathbf{k}}\sqrt{2E_{-\mathbf{k}}}a_{-\mathbf{k}}|0\rangle^*}{\langle 0|0\rangle^*} = -(2E_{\mathbf{k}})\alpha_{\mathbf{k}}\,.$$

Also, the expectation of *out* particle number in the squeezed out-state created by the *in* vacuum is [51] (and [50] for a recent review)

$$\begin{aligned}\langle N_{\mathbf{k}}\rangle &= \frac{|\alpha_{\mathbf{k}}|^2}{|1-|\alpha_{\mathbf{k}}|^2|}\,,\\ &= \frac{1}{4E^2_{\mathbf{k}}}\frac{|\mathcal{A}_{\mathbf{k}}|^2}{|1-|\frac{\mathcal{A}_{\mathbf{k}}}{2E_{\mathbf{k}}}|^2|}\,.\end{aligned}$$

From this we can write the total number of particles emitted as well as the total energy emitted

$$\frac{\bar{N}}{V_p} = \sum_{\mathbf{k}} \frac{1}{4E_{\mathbf{k}}^2} \frac{|\mathcal{A}_{\mathbf{k}}|^2}{|1 - |\frac{\mathcal{A}_{\mathbf{k}}}{2E_{\mathbf{k}}}|^2|}\,, \qquad \frac{\bar{E}}{V_p} = \sum_{\mathbf{k}} \frac{1}{4E_{\mathbf{k}}} \frac{|\mathcal{A}_{\mathbf{k}}|^2}{|1 - |\frac{\mathcal{A}_{\mathbf{k}}}{2E_{\mathbf{k}}}|^2|}\,. \tag{6.4}$$

It is understood in the previous formula that the sum is replaced by an integral for continous degrees of freedom. The other important aspect of this process is that the kinematics are completely similar to the closed string case. *Pair production of open strings has the same kinematics as coherent production of closed strings.* Indeed, the 2-point amplitude looks roughly like

$$\left\langle e^{iEX^0(\omega)} e^{iEX^0(0)} \right\rangle \approx \left\langle e^{2iEX^0(0)} g(E,\omega) \right\rangle\,, \tag{6.5}$$

where $g(E,\omega)$ is from the contraction and $\langle\rangle$ means the cylinder amplitude. Now, let us make the following ansatz for the 2-point 1-loop amplitude:

$$\mathcal{A} \approx \frac{1}{\sinh 2\pi E} w(E)\,, \tag{6.6}$$

where $w(E)$ is a polynomial and let us assume that it depends only weakly on the specific state considered. Note that this is exactly the form we found for our partial answer for the 1-point function with $E \to 2E$. At large energy, the square of this amplitude behaves like

$$\mathcal{A}^2 \sim e^{-4\pi E} \tag{6.7}$$

The density of state for the bosonic open string is

$$\begin{aligned} D(n) &\sim \frac{1}{\sqrt{2}} n^{-27/4} e^{4\pi\sqrt{n}} \\ E &= \sqrt{\mathbf{k}_{\parallel}^2 + n} \sim \sqrt{n} + \frac{\mathbf{k}_{\parallel}^2}{\sqrt{n}}\,. \end{aligned} \tag{6.8}$$

As in the case of closed string [35], the exponentials cancels exactly. This is not surprising. Emitting two open strings or one left/right identical closed string lead to the same energetic behavior.

There is still some difference as the phase space is now the world-volume of the brane and the final state is a squeezed state as opposed to a coherent state. Although we can neglect the extra term in the denominator in (6.4) we still have an extra factor of E in the denominator compared to the closed string case. Integrating over the world-volume dimensions of the brane will give an extra factor of $n^{1/4}$ per parallel direction, so the phase space is the opposite of the one for closed string emission. Hence, just from phase space considerations, higher dimensional branes have their open string production enhanced.

Altogether the results for our ansatz amplitude are

$$\bar{N} \propto N^2 g_s^2 \sum_n n^{(-31+p)/4} w(\sqrt{n})^2\,, \qquad \bar{E} \propto N^2 g_s^2 \sum_n n^{(-29+p/4)} w(\sqrt{n})^2\,. \tag{6.9}$$

It is important to note that the for the superstring case, the exponential again cancels but the power law will differ (as does the allowed value for p). Indeed for the superstring case we get (with $D(n) \approx n^{-11/4} e^{4\pi\sqrt{n}}$),

$$\bar{N} \propto N^2 g_s^2 \sum_n n^{(-15+p)/4} w(\sqrt{n})^2 \,, \qquad \bar{E} \propto N^2 g_s^2 \sum_n n^{(-13+p/4)} w(\sqrt{n})^2 \,. \tag{6.10}$$

At this point let us summarize the physical picture that we are getting:

- We can compare the average number of open strings emitted to what one get from closed strings emission at tree level [35],

  $$\bar{N} \propto \sum_n n^{-p/4-1} \,, \qquad \bar{E} \propto \sum_n n^{-p/4+2} \,. \tag{6.11}$$

  We see that this is very similar except that the phase space is inverted.

- The expressions (6.11) is divergent for $p = 0, 1, 2$ where it is believed that the back-reaction becomes important and one must regulates the answer. It was also argued in the litterature that we must always allow for the maximum phase space available (the D0 case). The argument is that the tachyon field should decay in a non-uniform way and that it should be possible to emit closed strings with momemtum parallel to the brane as well as perpendicular and therefore the phase space allowed for closed string production should correspond to the full nine spatial dimensions.

- For open strings, it is impossible for them to have momemtum in the perpendicular direction and therefore the phase space allowed, except for the D9 case, is suppressed compare to the closed string case. This seems to say that open string production would be negligible compare to closed string production because of phase space. Of course, it is possible that the unknown polynomial $w(\sqrt{n})$ makes the total number and total energy diverge for open strings on a D0-brane as well (to do so we need a strong energy dependence $w(\sqrt{n}) \approx n^{14}$ for the superstring case which appear unlikely).

- Finally, the emission of closed strings will be further enhanced by its own 1-loop contribution. This is very similar to the open strings amplitude except that we have two propagators. Naively, we expect that this extra propagator will suppress the amplitude by an extra factor of $1/E^2$ but more detailed calculation are needed here. Furthermore, there is also a 2-points tree level diagram that contributes to closed string production at order g_s. This diagram is not energetically supressed by propagators but neither is it enhanced by the number of spectators branes.

7. Conclusion

In this paper, we have laid down the very first step of a tedious but important calculation. We have shown that divergent terms appear in the amplitude when we expand it in terms of oscillator levels. We have also seen that unlike the tree level case, these terms cannot be 'gauged away'. Nevertheless, we have obtained partial results for the amplitude where we considered only a set of powers of q and summed over all oscillator levels and momenta. We have shown that by doing so, one can obtain a finite amplitude out of pieces that diverge when looked at individually. This is not suprising as from T-duality, we would expect that the winding number (momenta) and oscillator levels are interchangeable. We know that we need to sum over all momemtum before inverse Wick rotating in the tree level calculation and in this paper we have shown that the same is true (for at least a partial subset) for the oscillator mode in the 1-loop calculation.

Importantly we have also clarified where to perform the inverse Wick rotation. It is crucial to inverse Wick rotate only the final answer $\mathcal{A}(\tau)$ as this is the physicially meaningful quantity. Futhermore, the integral over the moduli space 's' and the sum over momentum and oscillators must be performed in the euclidean theory because all three operations converge only in that case.

We have also learned that the known techniques to do 1-loop amplitude are somehow insufficient to obtain the answer. Even, if we could succeed in finding the amplitude to the simplest state, what we are really interested in is the amplitude to any state and the calculation looks almost hopelessly complicated in that case. Considering the phenomenological importance of this calculation, we need more progress on this issue. We also confirmed that Sen's boundary state is really the correct boundary state and these ill-defined terms do not cause problem as in a any physical computation only finite results are obtained.

At this point, considering the form of the partial amplitude it is tempting to conjecture that the 1-point and 2-points open string production have the following form:

$$\mathcal{A}_{\rm 1pt}(E) \approx \frac{1}{\sinh \pi E} w(E)\,, \qquad \mathcal{A}_{\rm 2pts}(E) \approx \frac{1}{\sinh 2\pi E} w'(E)\,,$$

where $w(E)$ and $w'(E)$ are *polynomials* and most probably they are functions of the specific state considered as well. Indeed, unlike for the closed string case, there is no reason to expect that the amplitude is the same for all states at a given level. If this conjecture is true then one can show simply (if $w'(E)$ varies weakly for different states at large energy) that for the 2-point function the exponential suppression is exactly cancelled by the exponential growth of states. Then, and only then, can we have a production of open strings that is non-zero at high energy. Indeed, in the case where the exponential suppression is less then the Hagedorn exponential growth, we would have negligible open string production, and in the case where it is more we would have an exponentially divergent amplitude at large energy! Since the latter is unphysical and the former appears counter-intuitive as we expect the spectator branes to be excited, we believe this conjecture to be sensible. From here it then remains to find these unknown polynomials.

Although open strings production is not negligible by itself (it is not zero) it will be negligible when compared to closed strings production because of phase space consideration. Closed strings can go anywhere, in the bulk or the branes, and this enhances their production. There is a slight chance that the polynomial $w'(E)$ is just so such that the open string production does compete even after phase space consideration but this appears unlikely.

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A. SU(2) methods

A.1 Current algebra

In the main body of the text, we have evaluated the one-point function by expanding both the vertex operators and the boundary state in the Virasoro basis. As the boundary state is only partially known in this basis, only partial answers were obtained.

Another approach is possible that has the important advantage of considering the whole boundary state. We can use SU(2) current algebra or its fermionized version to express the vertex operators in an SU(2) invariant way and then use the SU(2) basis for the boundary state. This is possible as the boundary state only excites open strings with their momenta at an integer value of the SU(2) radius.

Indeed, the amplitude (4.2) can be written in the following way:

$$\begin{aligned} \mathcal{A}_{\mathrm{Eu}}(E) &= \int dx_0 \left\langle e^{-iEX^0} \right\rangle = 2\pi \sum_n \delta(E-n) \left\langle e^{-iEX^0_{\mathrm{osc}}} \right\rangle , \\ \mathcal{A}_{\mathrm{Eu}}(\tau) &= \sum_n \left\langle e^{-in\tau} e^{-inX^0_{\mathrm{osc}}} \right\rangle , \end{aligned}$$

where $\langle\rangle$ is the one-point function on the cylinder.

To evaluate $\left\langle e^{-in\tau} \right\rangle$ we can either write the vertex in terms of SU(2) current operators (J) or in terms of fermions. For the first case, the following representation of the boundary state seems appropriate

$$\begin{aligned} |B\rangle\!\rangle &= e^{-\lambda J_0^+} \, |N,\rangle\!\rangle \\ |B\rangle\!\rangle &= \sum_j \sum_{m \geq 0}^{j} \frac{(j+m)!}{(2m)!(j-m)!} (-\lambda)^{2m} \, . \, |j,m,m\rangle\!\rangle \end{aligned} \tag{A.1}$$

The SU(2) generators are defined by:

$$\begin{aligned} J^{\pm} &= \oint \frac{dz}{2\pi i} e^{\pm 2iX}(z) \\ J^3(z) &= \oint \frac{dz}{2\pi i} i\partial X(z) \, . \end{aligned} \tag{A.2}$$

As is usual we are interested in the affine lie algebra $\mathrm{SU}(2)_1$. We can express the vertex operator in terms of these operators for different SU(2) sectors (q) There is an extra factor of 2 from the Neumann boundary condition $X = X_L + X_R \to 2X_L$.

$$
\begin{array}{lll}
q=1 & e^{-iX}(z) & J^-(z) \\
q=2 & e^{-i2X}(z_1) & z_{12}^{-2}J^-(z_1)J^-(z_2) \\
\vdots & & \\
q=n & e^{-inX}(z_1) & \Pi_{i,j=1,i<j}^{n} z_{ij}^{-2}J^-(z_i)J^-(z_j)
\end{array}
$$

For a given sector, only one term in the exponential will contribute as the number of J^- and J^+ must match in order to have a non-zero amplitude. So the general amplitude for a given q sector will be proportinal to λ^q (this is the delta function of energy that we were getting previously). Furthermore, the J^- can be expanded in terms of modes

$$
J^-(z) = \sum_m J_m^- z^{-m} \,. \tag{A.3}
$$

As the amplitude should be independent of z, only a finite number of terms would contribute in the previous sum. The remaining computation then amounts to commuting all the SU(2) operators and acting on the SU(2) Ishibashi state. This is simply stated but becomes quite messy in practice.

A.2 Fermionization and path integral

Here let us just discuss two alternatives methods that might prove to be the key to making this calculation tractable. We do not give any details but merely indicate the general philosophy behind these two methods.

The $SU(2)_1$ system (or $SU(2)_2$ for superstring) allows for fermionic representations (alternatively SO(2) and SO(3)). This approach was used in [52] to calculate the vacuum amplitude and it was further generalized in [53] to the superstring case.

This approach has the distinct advantage of having the simplest version of the boundary state but as for the current algebra methods it quickly gets complicated for large momenta and the sum over 'n' appears to be intractable [54]. This method is similar in spirit to the previous calculation though the grassmanian nature of the variable simplifies the algebra.

Finally, as an alternative to the BCFT methods presented in this paper, one might wonder if one could just use directly path integral techniques to perform the calculation [37, 54]. This has been shown to be successful at tree-level and together with matrix techniques one can calculate the one-point function on the disc for various vertex operators. This is how [43] obtains the coefficients of the boundary state that we have used. Here the idea is to expand the exponential of the boundary action and to evaluate each correlation function on the cylinder. The results are Θ functions that then need to be integrated,

$$
\begin{aligned}
&\int dx^0 \left\langle e^{-iEX^0} e^{-\tilde{\lambda}\int d\tau e^{ix^0} e^{iX'(\tau)}} \right\rangle = \qquad\qquad (A.4)\\
&= \int dx^0 \int ds \sum_{n=0}^{\infty} e^{-iEx^0} \frac{(-\tilde{\lambda} e^{ix^0})^n}{n!} \int d\tau_1 \cdots \int d\tau_n \left\langle e^{-iEX^0} e^{-iX'(\tau_1)} \cdots e^{-iX'(\tau_n)} \right\rangle .
\end{aligned}
$$

This amplitude is simple to evaluate in general but the integrals over time are quite difficult

$$
\mathcal{A}(\tau) = \int ds \sum_{n=0}^{\infty} \frac{(-\tilde{\lambda} e^{i\tau})^n}{n!} (\mathrm{z.m.}) \int \mathrm{d}\tau_0 \cdots \mathrm{d}\tau_\mathrm{n} \prod_{\mathrm{i<j}}^{\mathrm{n}} \left|\Theta_1(\mathrm{i}\tau_{\mathrm{ij}}|\mathrm{is})\right|^2 \prod_{\mathrm{i=1}}^{\mathrm{n}} \left|\Theta_2(\mathrm{i}\tau_{0\mathrm{i}}|\mathrm{is})\right|^{-2\mathrm{n}} .
$$

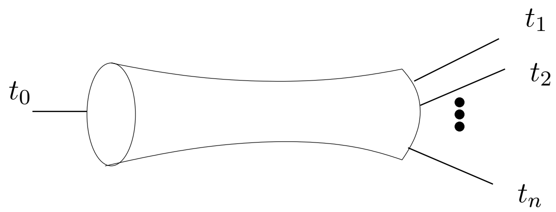

Figure 6: One point function where the brane anti-brane boundary is represented by n tachyons. The full amplitude is obtained by summing over all such diagrams with different n.

Here (z.m) is the zero mode part. All these methods are consistent in that they do not have any obvious simplifications and they quickly lead to quite complicated expressions. Hopefully, further research might find either an alternative method or some relations that allow us to complete this important calculation.

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